**SYNOPTIC: Slender Bodies of Minimum Drag in Hypersonic Viscous Flow,** H. Mirels and J. W. Ellinwood, The Aerospace Corporation, El Segundo, Calif.; *AIAA Journal,* Vol. 8, No. 10, pp. 1865–1869.

**Optimization, Hypersonic Flow, Viscid-Inviscid Flow Interactions**

### Theme

The thickness ratio that produces the minimum zero-lift drag on cones, wedges, and $\frac{3}{4}$ power law bodies is determined by application of the authors' viscous interaction theory for slender bodies in axisymmetric or plane hypersonic flow.

### Content

Earlier theories of drag minimization of slender axisymmetric shapes in hypersonic flow have neglected viscous interaction and transverse curvature effects, yet the optimal shapes so determined have been sufficiently slender to require both neglected effects. The recent development of a theory for axisymmetric viscid-inviscid interaction in the presence of a strong bow shock wave allows these effects to be included in the present treatment.

Let $C_D$ be the drag coefficient based on base area, $r_b$ the base radius, and $L$ the vehicle length. For a given perfect fluid with an isothermal, nonporous body surface, it is known that $C_DL^2/r_b^2$ is a function $\psi(\tilde{\Lambda})$ of a single variable $\tilde{\Lambda}$, where $\tilde{\Lambda}$ is $BL^{3/2}/r_b^2$ and

$$B = (1 + 3.46\, g_w)M_\infty(C\nu_\infty/u_\infty)^{1/2}$$

Here $g_w$ is the ratio of surface to stagnation enthalpy, $C$ is the Chapman-Rubesin constant, and $M_\infty$, $\nu_\infty$, and $u_\infty$ are freestream Mach number, kinetic viscosity, and velocity, respectively. Three constraint cases are considered: given base radius ($r_b$); given body volume ($V \propto L\, r_b^{\sigma+1}$), where $\sigma$ is 0 or 1 for plane or axisymmetric flow, respectively; or given surface area ($S \propto L\, r_b^{\sigma}$). For these three cases, respectively, the following expressions of drag $D$, where

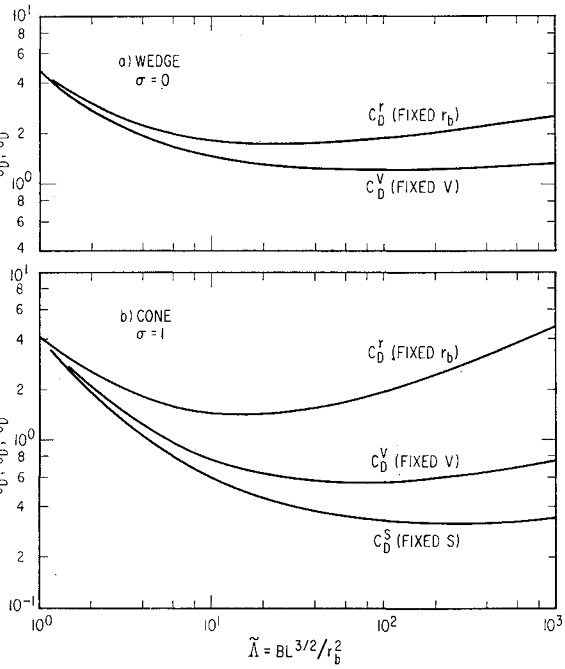


**Fig. 1 Variation of drag with $\tilde{\Lambda}$ for fixed $r_b$, $V$, or $S$; $g_w = 0$, $\gamma = 1.4$, Pr = 0.7: a) wedge, b) cone.**

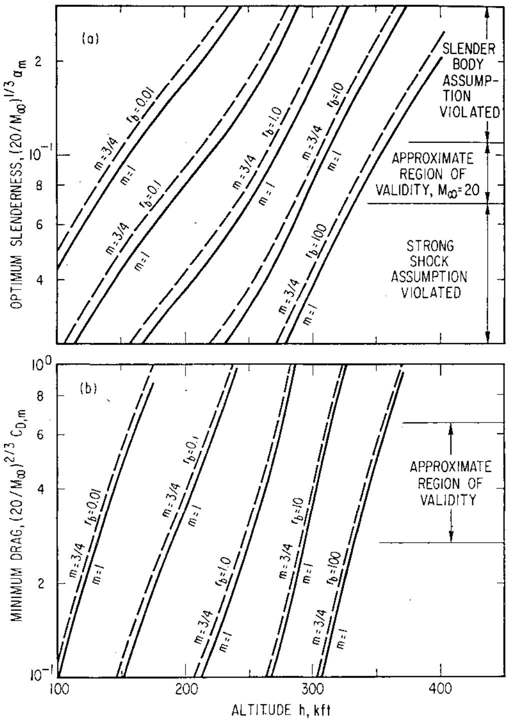


**Fig. 2 Effect of altitude on minimum drag body with fixed base $r_b$, ft; $C = 1$, $g_w = 0$, $\gamma = 1.4$, Pr = 0.7: a) optimum slenderness, b) minimum drag coefficient.**

$D \propto C_D r_b^{\sigma+1}$, are to be minimized:

$$D\, r_b^{-(1+3\sigma)/3} \propto \tilde{\Lambda}^{-4/3}\psi(\tilde{\Lambda}) \equiv C_D^r \qquad (1)$$

$$D\, V^{-(3\sigma+1)/(7+3\sigma)} \propto \tilde{\Lambda}^{-(6\sigma+10)/(3\sigma+7)}\psi(\tilde{\Lambda}) \equiv C_D^V \qquad (2)$$

$$D\, S^{-(3\sigma+1)/(3\sigma+4)} \propto \tilde{\Lambda}^{-(6\sigma+6)/(3\sigma+4)}\psi(\tilde{\Lambda}) \equiv C_D^S \qquad (3)$$

The variation of $C_D^r$, $C_D^V$, and $C_D^S$ with $\tilde{\Lambda}$, for cones and wedges, is shown in Fig. 1. The function $\psi$ approaches a constant (characteristic of inviscid flow) as $\tilde{\Lambda}$ vanishes. For strong interaction ($\tilde{\Lambda} \to \infty$), $\psi$ grows as $\tilde{\Lambda}^{(3+\sigma)/2}$, neglecting a possible logarithmic factor. It is seen that the minimum drag occurs at finite $\tilde{\Lambda}$ (except for the single case of $\sigma = 0$, given surface area). Thus, viscous interaction effects must be included when determining optimum thickness ratio for minimum drag.

Values of optimal $\tilde{\Lambda}$ in Fig. 1 range from moderate viscous interaction ($\tilde{\Lambda} \approx 15$) to relatively strong interaction ($\tilde{\Lambda} \approx 300$). The corresponding ratios of boundary-layer thickness to $r_b$ range from 0.6 to 7.5, and the ratios of friction drag to pressure drag range from about one to nearly 300.

The effect of altitude on optimum slenderness and corresponding $C_D$ is shown in Fig. 2 for the case of specified $r_b$. Results for cones ($m = 1$) and $\frac{3}{4}$ axisymmetric power law bodies ($m = \frac{3}{4}$) are given. The atmosphere assumed was

$$M_\infty \nu_\infty / u_\infty = E 10^{(h-350)/50}$$

where $1 \leq E \leq 2$ for $100 \leq h$ (kft) $\leq 400$. The optimum thickness ratio $\alpha_m$ is proportional to $M_\infty^{2/3}/Re_b^{1/3}$ where $Re_b$ is the flight Reynolds number referred to base radius. The validity of the theory is bounded at low altitudes by the assumption that the bow-shock is strong, and at high altitudes by the assumptions of no slip and no shock-layer merging. The results are useful primarily for $0.1 \leq r_b$(ft) $\leq 10.0$ at altitudes in the range $200 \leq h$(kft) $\leq 300$. Under these conditions the optimum slenderness, for minimum drag, is $\alpha_m = 0.1$ radians (approximately).

# Slender Bodies of Minimum Drag in Hypersonic Viscous Flow

HAROLD MIRELS* AND JOHN W. ELLINWOOD†
*The Aerospace Corporation, El Segundo, Calif.*

**Hypersonic viscous flow about slender cones, wedges, and $\frac{3}{4}$ power law bodies is considered. Body thickness ratios leading to minimum drag are determined subject to constraints of fixed base, fixed volume, or fixed surface area. It is assumed that the shocks are strong and that the fluid is ideal. It is found that minimum drag occurs in a flow region where viscous interaction and, for axisymmetric bodies, transverse curvature must be taken into account. For a given body shape and base radius, the optimum thickness ratio $\alpha_m$ is proportional to $M_\infty^{2/3}/Re_b^{1/3}$ where $M_\infty$ is the flight Mach number and $Re_b$ is the flight Reynolds number referenced to base radius. Numerical results are given for optimum thickness ratio and minimum drag as a function of altitude. The results are useful primarily for base radii between 0.1 and 10 ft at altitudes between $2 \times 10^5$ and $3 \times 10^5$ ft.**

## Nomenclature

$B$ = $(1 + 3.46\, g_w)M_\infty(CL/Re)^{1/2}$
$C$ = Chapman-Rubesin constant, $\mu_r T_\infty/\mu_\infty T_r$
$C_D$ = drag coefficient referenced to base area
$C_D{}^r$ = Eq. (3)
$C_D{}^V$ = Eq. (5)
$C_D{}^S$ = Eq. (7a)
$g_w$ = $T_w/T_O$
$h$ = altitude
$L$ = vehicle length
$M_\infty$ = freestream Mach number $u_\infty/a_\infty$
$m$ = power law exponent $r_w \sim x^m$
$Pr$ = Prandtl number
$Re$ = Reynolds number $\rho_\infty u_\infty L/\mu_\infty$
$Re_b$ = $\rho_\infty u_\infty r_b/\mu_\infty$
$r$ = transverse distance
$r_w(x), r_b$ = body ordinate; value at base
$S$ = surface area (symmetric body assumed for $\sigma = 0$ case)
$T$ = temperature
$T_O$ = freestream stagnation temperature
$u_\infty$ = freestream velocity
$V$ = volume (symmetric body assumed for $\sigma = 0$ case)
$x$ = axial distance
$\alpha$ = $r_b/L$
$\alpha_e$ = $(r_b + \delta_b^*)/L$
$\alpha_m{}^r$ = Eq. (4)
$\alpha_m{}^V$ = Eq. (6)
$\alpha_m{}^S$ = Eq. (7b)
$\gamma$ = ratio of specific heats
$\delta_b^*$ = boundary-layer displacement thickness at base
$\bar{\chi}$ = $(1 + 3.46\, g_w)(M_\infty/\alpha^2)(C/Re)^{1/2}$
$\mu$ = viscosity
$\rho$ = density
$\sigma$ = 0, 1 for two-dimensional or axisymmetric bodies, respectively
$\psi$ = $C_D/\alpha^2$

***Subscripts***

$m$ = minimum drag value
$\infty$ = freestream value

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* Head, Aerodynamics and Heat Transfer Department, Laboratories Operations. Associate Fellow AIAA.
† Member of the Technical Staff, Laboratories Operations. Associate Fellow AIAA.

## I. Introduction

THE determination of body shapes with minimum drag subject to certain constraints is of importance in aerodynamic design. A recent survey of activities in this area is given in Ref. 1.

Consider the following problem: given the base radius, volume, or surface area of a plane or axisymmetric body in a hypersonic flow, what forebody shape will yield minimum drag? This problem does not appear to have been treated in the literature with a realistic consideration of boundary-layer effects. For instance, in Ref. 1, viscous interaction and transverse curvature effects are neglected, and a Newtonian pressure distribution and a constant local skin-friction coefficient are assumed. Furthermore, the local skin-friction coefficient is assumed to be known, a priori, and its dependence on the local pressure is ignored. It is shown herein that viscous interaction and (for axisymmetric bodies) transverse curvature effects need to be taken into account. In addition, the effect of the local pressure on the local shear also needs to be considered. Hence, the results of Ref. 1 appear to be unrealistic.

In the present paper, hypersonic flow about slender cones, wedges, and $\frac{3}{4}$ power law bodies is considered. Body thickness ratios leading to minimum drag are determined subject to constraints of fixed base, volume, or surface area. The results are based on the viscous interaction theory of Refs. 2 and 3.